The Quantum Overfitting Amplification Theorem and Heavy-Tail Quantum-Adjusted Deflated Sharpe Ratio (Q-DSR$_\nu$)

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Abstract. Quantum search and amplitude amplification algorithms [1–3] provide an optimal quadratic query speedup over unstructured classical search, scanning $M_{\text{quant}}(C) = \lfloor (\kappa C)^2 \rfloor$ candidate hypotheses with only C coherent oracle evaluations. While celebrated in combinatorial optimization and quantum machine learning (QML), we prove that applying quadratic quantum search to model selection, hyperparameter tuning, or trading-strategy backtesting on a **fixed finite historical sample** induces a sharp, previously unformalized statistical pathology: **Quantum Overfitting Amplification**. Under the complete null hypothesis H_0 (zero true out-of-sample signal), we establish four rigorous theorems: **(1) Universal Sub-Weibull Amplification Law (Theorem 1):** for any null score distribution with sub-Weibull right tail $-\ln \bar{F}(t) = c\,t^\alpha + o(t^\alpha)$ $(\alpha > 0)$, fixing the oracle query budget C leaves expected out-of-sample performance identically zero while amplifying the expected in-sample false discovery by the universal asymptotic factor $2^{1/\alpha}$—strictly independent of the algorithm's constant κ—yielding $\sqrt{2} \approx 1.414\times$ for Gaussian tails ($\alpha = 2$) and $2.000\times$ for Laplace/exponential tails ($\alpha = 1$). **(2) Polynomial Heavy-Tail Explosion (Theorem 2):** in the Fréchet maximum domain of attraction $\bar{F}(t) \sim c_\nu\, t^{-\nu}$ ($\nu > 1$, e.g., Student-t_ν financial or genomic statistics), the constant factor breaks down into an unbounded **polynomial explosion** $\mathbb{E}[X^*_{\text{quant}}(C)]/\mathbb{E}[X^*_{\text{cl}}(C)] \sim \kappa^{2/\nu} C^{1/\nu}$, reaching $11.76\times$ at $C = 1{,}000$ for Student-t_3. **(3) Collapse of C-Indexed Multiple Testing and Exact Q-DSR$_\nu$ (Theorem 3):** any multiple-testing procedure (Bonferroni, Šidák, or Bailey–López de Prado's Deflated Sharpe Ratio) calibrated on the quantum query count C suffers asymptotic collapse $\text{FPR}(C) \to 1$, and even Gaussian quantum-adjusted corrections fail at 100% FPR on heavy-tailed distributions. We derive the exact closed-form **Quantum-Adjusted Deflated Sharpe Ratio (Q-DSR)** and its heavy-tail extension $\text{Q-DSR}_\nu(x; C) = [F_{t_\nu}(x)]^{M_{\text{eff}}^{\text{quant}}}$, proving exact finite-sample control $\text{FPR} = \alpha$ under independence and guaranteed conservative control $\text{FPR} \le \alpha$ for arbitrary non-negatively correlated catalogs via Slepian's inequality—while demonstrating empirically why naive participation-ratio discounting is dangerously anti-conservative (FPR up to 56.9%). **(4) Landauer Thermodynamic Reconciliation (Theorem 4):** although the C coherent Grover iterations are unitarily reversible (zero intrinsic Landauer dissipation), extracting and resetting the winning index register of $q = \lceil \log_2 M_{\text{quant}} \rceil$ qubits obeys the Sagawa–Ueda/Landauer lower bound $Q^{\text{quant}}_{\min}(C) = k_B T_{\text{th}} \ln M_{\text{quant}}(C) \sim 2\, k_B T_{\text{th}} \ln C$, exactly **twice** the minimal classical selection dissipation at equal query budget C while conserving $k_B T_{\text{th}} \ln 2$ per selection bit. Exact unitary statevector Grover simulations ($q \le 12$, unitarity error $\le 3.6 \times 10^{-15}$) and large-scale extreme-value benchmarks ($R = 4{,}000$ replicates, $C \in [10, 1{,}000]$) validate all four theorems to high precision.

Keywords: Quantum search; Grover's algorithm; Quantum machine learning; Backtest overfitting; Extreme value theory; Sub-Weibull tails; Heavy tails; Deflated Sharpe Ratio; Landauer's principle.

1. Introduction and Prior-Art Novelty Audit

1.1. The Paradox Between Computational Speedup and Statistical Immutability

In the delayed-choice quantum eraser, sorting fixed detection clicks *a posteriori* on the signal screen D_0 creates no new causal information, yet reveals conditional sub-patterns whose true physical cost is governed by the denominator of rejected trials (PostBQP = PP [4]). Strictly analogously, in machine learning and quantitative finance, selecting the best-performing model among M candidates evaluated on a fixed historical dataset $\mathcal{D}_{\text{IS}}$ constitutes **post-selection on the past**.

Quantum information theory establishes two celebrated results that have thus far been analyzed separately:

1. **Quadratic computational advantage:** the search algorithm of Grover [1], the minimum/maximum-finding procedure of Dürr and Høyer [2], and amplitude amplification [3] reduce the oracle query complexity of identifying the best candidate among M hypotheses from $\Theta(M)$ to $\Theta(\sqrt{M})$—a bound proved asymptotically optimal by Bennett et al. [5].

2. **Invariance of statistical sample complexity:** Arunachalam and de Wolf [6] prove that quantum PAC sample complexity remains asymptotically identical to the classical bound $\Theta((d + \log(1/\delta))/\varepsilon)$, alongside the fundamental informational limits imposed by the Holevo [7] bound and the barren-plateau vs. classical-simulability dilemma [8]. By independence between the fixed historical sample $\mathcal{D}_{\text{IS}}$ and unseen future data $\mathcal{D}_{\text{OOS}}$, no unitary computation on $\mathcal{D}_{\text{IS}}$ can improve out-of-sample generalization when the true underlying signal is zero.

The central question of this paper emerges from the direct collision of these two principles: *at a fixed budget of C oracle queries, what happens when a practitioner accelerates model selection or strategy backtesting using quantum search?* Far from enhancing genuine scientific discovery, the quadratic speedup $M_{\text{quant}}(C) = \Theta(C^2)$ converts entirely into a **systematic amplification of in-sample false positives**, whose exact law depends sharply on the score tail regime (sub-Weibull vs. Fréchet/heavy tails), collapses every multiple-testing procedure indexed on the query budget C, and shifts the entire Landauer thermodynamic dissipation from intermediate query iterations onto final measurement and register reset.

1.2. Verified Prior-Art Novelty Audit

Targeted literature searches across arXiv and peer-reviewed venues (combining *quantum overfitting amplification*, *Quantum-Adjusted Deflated Sharpe Ratio*, *Q-*

DSR, and Grover search with multiple testing, *backtest overfitting*, extreme value theory, Student-t distributions, and Landauer's principle) confirm that no prior publication establishes the quantitative synthesis or the four theorems proved here. Table 1 delineates the boundary between established prior art and our original theoretical contributions.

2. Formal Framework at Fixed Query Budget C

Let $\mathcal{D}_{\mathrm{IS}} = (Z_1, \ldots, Z_T)$ be a fixed historical sample of size T and $\mathcal{D}_{\mathrm{OOS}} = (Z_{T+1}, \ldots, Z_{2T})$ an independent future sample. Consider a catalog of candidate hypotheses, models, or strategies $\mathcal{H} = \{h_1, \ldots, h_{M_{\max}}\}$ equipped with an empirical evaluation statistic $\mathcal{S}(h_m; \mathcal{D}) \in \mathbb{R}$ (normalized Sharpe ratio, t-statistic, correlation score, or penalized log-likelihood).

Complete Null Hypothesis (H_0). For every $m \in \{1, \ldots, M_{\max}\}$, the in-sample scores $X_m := \mathcal{S}(h_m; \mathcal{D}_{\mathrm{IS}})$ share a continuous cumulative distribution function F (survival function $\bar{F}(t) := 1 - F(t)$) satisfying $\mathbb{E}[X_m] = 0$ (unless stated otherwise, $X_1, \ldots, X_{M_{\max}}$ are independent; the correlated case with covariance matrix Σ is addressed explicitly in Theorem 3, Part 4). The out-of-sample scores $Y_m := \mathcal{S}(h_m; \mathcal{D}_{\mathrm{OOS}})$ satisfy $\mathbb{E}[Y_m] = 0$ and are independent of $\mathcal{D}_{\mathrm{IS}}$ and of all internal algorithmic randomness.

Search Models at Fixed Oracle Query Budget $C \in \mathbb{N}^*$. One call to the empirical oracle $\mathcal{O}_{\mathcal{S}}$ evaluates $\mathcal{S}(h_m; \mathcal{D}_{\mathrm{IS}})$ (classically on a single index m, or coherently via $|m\rangle|0\rangle \mapsto |m\rangle|\mathcal{S}(h_m; \mathcal{D}_{\mathrm{IS}})\rangle$ across a superposition of indices). At a fixed budget of C oracle calls:

1. **Classical search ($\mathcal{A}_{\mathrm{cl}}$):** queries at most $M_{\mathrm{cl}}(C) = C$ distinct candidates and outputs $\hat{m}_{\mathrm{cl}} = \arg\max_{1 \le m \le C} X_m$, with maximum score $X^*_{\mathrm{cl}}(C) := \max_{1 \le m \le C} X_m$.

2. **Quantum amplitude-amplification search ($\mathcal{A}_{\mathrm{quant}}$):** deploys thresholded Grover search [1, 3] or the quantum maximum-finding algorithm of Dürr and Høyer [2] to isolate with probability $1 - \delta$ the maximum over an effective hypothesis space of cardinality

$$M_{\mathrm{quant}}(C) = \lfloor (\kappa\, C)^2 \rfloor\,, \qquad X^*_{\mathrm{quant}}(C) := \max_{1 \le m \le M_{\mathrm{quant}}(C)} X_m, \tag{1}$$

where $\kappa > 0$ is the algorithmic efficiency constant: $\kappa_{\mathrm{Grover}} = 4/\pi \approx 1.27324$ for Grover amplification on a unique maximum in $C = \lfloor \frac{\pi}{4}\sqrt{M} \rfloor$ queries (success probability $\sin^2((2C+1)\arcsin(1/\sqrt{M})) = 1 - \mathcal{O}(1/M)$), and $\kappa_{\mathrm{DH}} = 1/c_{\mathrm{DH}} > 0$ for unthresholded Dürr–Høyer search (expected query count $c_{\mathrm{DH}}\sqrt{M}$).

The **quantum overfitting amplification factor** at query budget C is defined as

$$\mathcal{R}(C) := \frac{\mathbb{E}\big[X^*_{\mathrm{quant}}(C)\big]}{\mathbb{E}\big[X^*_{\mathrm{cl}}(C)\big]}. \tag{2}$$

3. Theoretical Results: Four Fundamental Theorems

3.1. Theorem 1 — Universal Amplification Law Under Sub-Weibull Tails

Theorem 1 (Universal $2^{1/\alpha}$ Amplification Under Sub-Weibull Tails)**.** *Let $X_1, X_2, \ldots$ be i.i.d. integrable random variables ($\mathbb{E}[|X_1|] < \infty$) under H_0 whose survival function $\bar{F}(t) = \mathbb{P}(X_1 > t)$ satisfies the exact sub-Weibull tail condition of index $\alpha > 0$ and scale $c > 0$ [10]:*

$$-\ln \bar{F}(t) = c\, t^{\alpha} + o(t^{\alpha}) \qquad (t \to +\infty). \tag{3}$$

Then, for any algorithmic constant $\kappa > 0$ setting $M_{\mathrm{quant}}(C) = \lfloor (\kappa C)^2 \rfloor$ against $M_{\mathrm{cl}}(C) = C$:

1. *(Out-of-sample invariance) $\mathbb{E}[Y_{\hat{m}_{\mathrm{quant}}}] = \mathbb{E}[Y_{\hat{m}_{\mathrm{cl}}}] = 0$ identically for all $C \ge 1$.*

2. *(Universal in-sample amplification) Both in expectation and in probability,*

$$\lim_{C \to \infty} \mathcal{R}_{\mathrm{subW}}(C) = \lim_{C \to \infty} \frac{\mathbb{E}[X^*_{\mathrm{quant}}(C)]}{\mathbb{E}[X^*_{\mathrm{cl}}(C)]} = 2^{1/\alpha}, \qquad \frac{X^*_{\mathrm{quant}}(C)}{X^*_{\mathrm{cl}}(C)} \xrightarrow[C \to \infty]{\mathbb{P}} 2^{1/\alpha}. \tag{4}$$

*This limit is **strictly independent of** κ and of the scale c. In particular: Gaussian tails ($\alpha = 2$) $\Rightarrow \sqrt{2} \approx 1.4142$ (+41.4%); Laplace/exponential tails ($\alpha = 1$) $\Rightarrow 2.0000$ (+100.0%); stretched-exponential tails ($\alpha = 1/2$) $\Rightarrow 4.0000$ (+300.0%).*

Proof. **Step 1 (Out-of-sample invariance).** The selected index $\hat{m}_{\mathrm{quant}}$ is measurable with respect to $(\mathcal{D}_{\mathrm{IS}}, \xi)$, where ξ denotes the internal quantum measurement randomness, independent of $\mathcal{D}_{\mathrm{OOS}}$. By the law of iterated expectations,

$$\mathbb{E}[Y_{\hat{m}_{\mathrm{quant}}}] = \mathbb{E}\big[\mathbb{E}[Y_{\hat{m}_{\mathrm{quant}}} \mid \mathcal{D}_{\mathrm{IS}}, \xi]\big] = \sum_m \mathbb{P}(\hat{m}_{\mathrm{quant}} = m)\, \mathbb{E}[Y_m] = 0.$$

This agrees with the quantum PAC bounds of Arunachalam and de Wolf [6]: coherent oracle queries on a fixed historical sample create zero statistical information about an independent future sample.

Step 2 (L^1 and in-probability asymptotics of the maximum). Define the $(1 - 1/M)$-quantile $u_M := \inf\{t \in \mathbb{R} : \bar{F}(t) \le 1/M\}$. By continuity of F, $-\ln \bar{F}(u_M) = \ln M$, and asymptotic inversion of (3) yields

$$u_M = \left(\frac{\ln M}{c}\right)^{1/\alpha} \big(1 + o(1)\big) \qquad (M \to \infty). \tag{5}$$

Set $X_{(M)} := \max_{1 \le m \le M} X_m$. *Lower bound:* for any $\varepsilon \in (0, 1)$, let $t_- := (1 - \varepsilon)u_M$. By (3) and (5), $-\ln \bar{F}(t_-) = (1 - \varepsilon)^{\alpha} \ln M \cdot (1 + o(1))$, so $M\bar{F}(t_-) = M^{1 - (1-\varepsilon)^{\alpha} + o(1)} \to +\infty$ and $\mathbb{P}(X_{(M)} \le t_-) = (1 - \bar{F}(t_-))^M \le \exp(-M\bar{F}(t_-)) \to 0$. Since $X^-_{(M)} \le X^-_1$ with $\mathbb{E}[X^-_1] < \infty$, we have $\mathbb{E}[X_{(M)}] \ge t_- \mathbb{P}(X_{(M)} > t_-) - \mathbb{E}[X^-_1]$, hence $\liminf_{M \to \infty} \mathbb{E}[X_{(M)}]/u_M \ge 1 - \varepsilon$. *Upper bound:* for any $\varepsilon > 0$, set $t_+ := (1 + \varepsilon)u_M$ and

Table 1. Prior-art comparison against verified literature: exact boundary between established results and the original theoretical contributions of this paper.

Domain / Topic	Established Prior Art (Verified References)	Original Contribution of This Paper
Quantum Search & Complexity	Quadratic $\mathcal{O}(\sqrt{M})$ speedup of Grover [1], Dürr and Høyer [2], and Brassard et al. [3]; $\Omega(\sqrt{M})$ optimality [5]; PAC sample-complexity invariance [6].	Formal coupling with extreme value theory at a **fixed oracle query budget** C: exact quantification of the overfitting amplification ratio $\mathbb{E}[X^*_{\text{quant}}(C)]/\mathbb{E}[X^*_{\text{cl}}(C)]$.
Extreme Value Theory & Backtest Overfitting	Gumbel/Fréchet maximum domains of attraction [9]; sub-Weibull tail distributions [10]; classical *Deflated Sharpe Ratio* and PBO [11, 12].	**Theorem 1:** universal $2^{1/\alpha}$ amplification law (independent of κ) under sub-Weibull tails. **Theorem 2:** unbounded polynomial explosion $\kappa^{2/\nu}C^{1/\nu}$ under Fréchet / Student-t_ν heavy tails.
Multiple Testing & Correlation Control	Bonferroni, Šidák, and classical DSR indexed on classical trial count; Slepian [13] inequality for maxima of correlated Gaussian processes.	**Theorem 3:** proof of exponential $\text{FPR}(C) \to 1$ collapse for any C-indexed test, failure of Gaussian Q-DSR under heavy tails, exact derivation of Q-DSR$_\nu$, and proof of the anti-conservative participation-ratio trap vs. Slepian.
Information Thermodynamics	Landauer [14] principle, reversible computation [15], measurement–erasure thermodynamics [16]; classical $k_BT\ln N$ cost of *Best-of-N*.	**Theorem 4:** exact reconciliation between unitary reversibility of the C Grover iterations (0 intrinsic dissipation) and the $2\times$ factor paid at measurement/reset ($\sim 2k_BT_{\text{th}}\ln C$), plus the heavy-tail energy–mirage scaling law.

choose $\delta \in (0,1)$ such that $\eta := (1-\delta)(1+\varepsilon)^\alpha - 1 > 0$. By (3), there exists $T_0 > 0$ such that $\bar{F}(t) \le e^{-c't^\alpha}$ for all $t \ge T_0$ with $c' := c(1-\delta)$. For sufficiently large M ($t_+ \ge T_0$), the union bound gives

$$\mathbb{E}[X_{(M)}] \le \mathbb{E}[X^+_{(M)}] \le t_+ + M\int_{t_+}^{\infty} e^{-c't^\alpha}\,dt.$$

Changing variables via $s = c't^\alpha$ ($s_+ := c't_+^\alpha = (1+\eta+o(1))\ln M$), the upper incomplete Gamma tail satisfies $\int_{s_+}^\infty s^{1/\alpha-1}e^{-s}\,ds = \mathcal{O}(s_+^{1/\alpha-1}e^{-s_+})$, whence

$$M\int_{t_+}^{\infty} e^{-c't^\alpha}\,dt = \mathcal{O}\Big(M\,u_M^{1-\alpha}\,M^{-(1+\eta+o(1))}\Big) = o(1).$$

Thus $\limsup_{M\to\infty}\mathbb{E}[X_{(M)}]/u_M \le 1+\varepsilon$. Since $\varepsilon > 0$ is arbitrary, $\mathbb{E}[X_{(M)}]/u_M \to 1$ and $X_{(M)}/u_M \xrightarrow{\mathbb{P}} 1$.

Step 3 (Quantum-to-classical ratio). With $M_{\text{cl}}(C) = C$ and $\ln M_{\text{quant}}(C) = 2\ln C + 2\ln\kappa + o(1)$, Eq. (5) gives

$$\begin{aligned}\mathcal{R}_{\text{subW}}(C) &= \left(\frac{\ln M_{\text{quant}}(C)}{\ln C}\right)^{1/\alpha}\big(1+o(1)\big)\\ &= \left(2+\frac{2\ln\kappa}{\ln C}\right)^{1/\alpha}\big(1+o(1)\big) \xrightarrow[C\to\infty]{} 2^{1/\alpha}.\ \square\end{aligned}$$

Corollary 1.1 (Pre-Asymptotic Approach From Above)**.** *For Grover's algorithm ($\kappa = 4/\pi > 1$, so $2\ln\kappa \approx +0.4831 > 0$), the factor $(2+\frac{2\ln(4/\pi)}{\ln C})^{1/\alpha}$ converges to $2^{1/\alpha}$* ***from above*** *(in the Gaussian case, the sub-leading $-\ln\ln M/(4\ln M)$ term further amplifies this positive finite-C offset). Consequently, at practical budgets $C \in [10, 1{,}000]$, empirical amplification exceeds the asymptote ($1.53\times$ vs. $\sqrt{2} = 1.41\times$ for Gaussian tails and $2.08\times$ vs. $2.00\times$ for Laplace tails at $C = 1{,}000$; cf. Section 4).*

3.2. Theorem 2 — Polynomial Explosion Under Fréchet Heavy Tails

Daily financial returns, genomic test statistics, and small-sample Student-t statistics ($T = \nu + 1$) belong to the **Fréchet maximum domain of attraction** with power-law tails, where the constant bound $2^{1/\alpha}$ of Theorem 1 breaks down completely.

Theorem 2 (Polynomial Explosion Under Fréchet Heavy Tails)**.** *Let $X_1, X_2, \ldots$ be i.i.d. random variables under H_0 whose survival function is regularly varying with index $-\nu$, $\nu > 1$ (integrability condition $\mathbb{E}[X_1^+] < \infty$):*

$$\bar{F}(t) = \mathbb{P}(X_1 > t) \sim c_\nu\, t^{-\nu} \qquad (t \to +\infty,\ c_\nu > 0). \quad (6)$$

(For a Student-t_ν law, the exact tail constant is $c_\nu = \frac{\Gamma((\nu+1)/2)\,\nu^{\nu/2-1}}{\sqrt{\pi}\,\Gamma(\nu/2)}$.) At fixed query budget C with $M_{\text{cl}}(C) = C$ and $M_{\text{quant}}(C) = \lfloor(\kappa C)^2\rfloor$:

1. *$\mathbb{E}[Y_{\hat{m}_{\text{quant}}}] = 0$ identically.*
2. *The in-sample mirage undergoes an* ***unbounded polynomial explosion in*** *$C^{1/\nu}$:*

$$\begin{aligned}\mathcal{R}_{\text{Fréchet}}(C) &= \frac{\mathbb{E}[X^*_{\text{quant}}(C)]}{\mathbb{E}[X^*_{\text{cl}}(C)]}\\ &= \kappa^{2/\nu}\,C^{1/\nu}\,\big(1+o(1)\big). \qquad (7)\end{aligned}$$

For Grover's algorithm ($\kappa = 4/\pi$),

$$\mathcal{R}^{\text{Grover}}_{\text{Fréchet}}(C) \sim \left(\frac{4}{\pi}\right)^{2/\nu} C^{1/\nu}, \qquad (8)$$

with both $c_\nu^{1/\nu}$ and $\Gamma(1-1/\nu)$ cancelling identically. (For $0 < \nu \le 1$, where $\mathbb{E}[X_1^+] = +\infty$, the exact same scaling law $\kappa^{2/\nu}C^{1/\nu}$ governs the ratio of medians and of every fixed quantile.)

Proof. From (6), the normalizing quantile $a_M := \bar{F}^{-1}(1/M)$ satisfies $a_M = (c_\nu M)^{1/\nu}(1+o(1))$. By the Fisher–Tippett–Gnedenko theorem in the Fréchet domain [9, Thm. 3.3.7], $X_{(M)}/a_M \xrightarrow{d} \Phi_\nu$, with distribution $\mathbb{P}(\Phi_\nu \le x) = \exp(-x^{-\nu})$ ($x > 0$). For $\nu > 1$, picking $r \in (1,\nu)$ ensures $\mathbb{E}[(X_1^+)^r] < \infty$, guaranteeing uniform integrability of $\{X^+_{(M)}/a_M\}_{M\ge1}$ (while

$\mathbb{E}[X_{(M)}^-] \le \mathbb{E}[X_1^-] = o(a_M)$), hence convergence of expectations:

$$\lim_{M\to\infty} \frac{\mathbb{E}[X_{(M)}]}{a_M} = \mathbb{E}[\Phi_\nu] = \int_0^\infty \nu x^{-\nu} e^{-x^{-\nu}}\, dx \overset{u=x^{-\nu}}{=} \int_0^\infty u^{-1/\nu} e^{-u}\, du = \Gamma\Big(1-\frac{1}{\nu}\Big), \tag{9}$$

yielding the exact asymptotic equivalence

$$\mathbb{E}[X_{(M)}] = \Gamma\Big(1-\frac{1}{\nu}\Big)\, c_\nu^{1/\nu}\, M^{1/\nu}\,\big(1+o(1)\big). \tag{10}$$

Taking the ratio between $M_{\text{quant}}(C) = \kappa^2 C^2(1+o(1))$ and $M_{\text{cl}}(C) = C$ cancels $\Gamma(1-1/\nu)$ and $c_\nu^{1/\nu}$ identically, proving (7). □

Remark 2.1 (Statistical Caveat on Self-Normalization). Theorem 2 holds exactly for: (i) small-sample t-statistics ($T = \nu + 1$) under Gaussian returns (exact Student-t_ν null law), and (ii) cumulative return scores normalized by a fixed reference volatility $\sum_{t=1}^T r_{m,t}/(\sigma_0\sqrt{T})$ when returns $r_{m,t}$ are regularly varying of index ν (by the single-big-jump principle, the finite sum preserves exact tail index ν). By contrast, a purely self-normalized Sharpe ratio $\sqrt{T}\bar{r}/\hat{s}$ over T periods is deterministically bounded on $[-\sqrt{T}, +\sqrt{T}]$ by the Cauchy–Schwarz inequality and exhibits Fréchet scaling only in the intermediate regime $X^* \ll \sqrt{T}$ prior to saturation.

3.3. Theorem 3 — Collapse of C-Indexed Multiple Testing and Exact Derivation of Q-DSR and Q-DSR$_\nu$

Calibrating any multiple-testing correction (Bonferroni, Šidák, or the *Deflated Sharpe Ratio* of Bailey and López de Prado [11]) on the **quantum query count** C rather than the effective superposition cardinality causes a catastrophic breakdown of Type-I error control.

Theorem 3 (Collapse to FPR → 1 and Exact Restoration via Q-DSR / Q-DSR$_\nu$)**.** *Under H_0, consider a level-α test ($\alpha \in (0,1)$) applied to the quantum winner $X^*_{\text{quant}}(C) = \max_{1\le m\le M_{\text{quant}}(C)} X_m$ with $M_{\text{quant}}(C) = \lfloor(\kappa C)^2\rfloor$ ($\kappa > 0$).*

1. *(Exponential collapse of every C-indexed test) Let $\tau_\alpha(C)$ be the threshold calibrated at nominal level α for C independent trials via Šidák ($\bar{F}(\tau_\alpha(C)) = 1-(1-\alpha)^{1/C}$) or Bonferroni ($\bar{F}(\tau_\alpha(C)) = \alpha/C$). Its actual false-positive rate on $X^*_{\text{quant}}(C)$ converges **exponentially fast to** 1:*

$$\begin{aligned}\text{FPR}_{\text{Šidák}(C)}(C) &= 1-(1-\alpha)^{\lfloor(\kappa C)^2\rfloor/C} \\ &= 1-\exp\big(-\kappa^2|\ln(1-\alpha)|\,C + \mathcal{O}(1)\big) \xrightarrow[C\to\infty]{} 1,\end{aligned} \tag{11}$$

$$\begin{aligned}\text{FPR}_{\text{Bonf}(C)}(C) &= 1-\Big(1-\frac{\alpha}{C}\Big)^{\lfloor(\kappa C)^2\rfloor} \\ &\ge 1-\exp\big(-\alpha\kappa^2 C + \mathcal{O}(1)\big) \xrightarrow[C\to\infty]{} 1.\end{aligned} \tag{12}$$

Likewise, for the naive DSR of Bailey and López de Prado [11] indexed on C ($\widehat{\text{SR}}_0(C) + z_{1-\alpha}$ on standardized Gaussian scores), the gap $(\sqrt{2}-1)\sqrt{2\ln C} - z_{1-\alpha} \to +\infty$ forces $\lim_{C\to\infty}\text{FPR}_{\text{DSR}(C)}(C) = 1$.

2. *(Failure of Gaussian Q-DSR under heavy tails) Applying a Gaussian quantum-adjusted threshold (even when indexed on the true cardinality $M_{\text{quant}}(C)$: $\tau_{\text{G}} = \mathcal{O}(\sqrt{\ln C})$) to Laplace scores ($\alpha = 1$, $X^*_{\text{quant}} = \Theta(\ln C)$) or Student-$t_\nu$ scores ($X^*_{\text{quant}} = \Theta(C^{2/\nu})$) still yields $\lim_{C\to\infty}\text{FPR}_{\text{Gauss-QDSR}}(C) = 1$.*

3. *(Exact closed-form Q-DSR and Q-DSR$_\nu$) Define $M^{\text{quant}}_{\text{eff}}(C) := \min(M_{\text{catalog}}, \lfloor(\kappa C)^2\rfloor)$. In the Gaussian / sub-Gaussian regime (direct quantum extension of Bailey and López de Prado [11], with Euler–Mascheroni constant $\gamma \approx 0.57721566$):*

$$\widehat{\text{SR}}_0^{\text{Q}}(C) := \sqrt{V[\widehat{\text{SR}}]}\Big[(1-\gamma)\,\Phi^{-1}\Big(1-\frac{1}{M^{\text{quant}}_{\text{eff}}}\Big) + \gamma\,\Phi^{-1}\Big(1-\frac{1}{e\,M^{\text{quant}}_{\text{eff}}}\Big)\Big], \tag{13}$$

$$\text{Q-DSR}(\widehat{\text{SR}}) := \Phi\left(\frac{\big(\widehat{\text{SR}} - \widehat{\text{SR}}_0^{\text{Q}}(C)\big)\sqrt{T-1}}{\sqrt{1-\hat{\gamma}_3\,\widehat{\text{SR}} + \frac{\hat{\gamma}_4-1}{4}\,\widehat{\text{SR}}^2}}\right). \tag{14}$$

*In the exact regime (Gaussian or heavy-tailed Student-t_ν, $\nu > 0$), the **Exact Heavy-Tail Quantum-Adjusted Deflated Score** is*

$$\begin{aligned}\text{Q-DSR}_\nu(x;C) &:= \big[F_{t_\nu}(x)\big]^{M^{\text{quant}}_{\text{eff}}(C)} \\ &= \exp\Big(M^{\text{quant}}_{\text{eff}}(C)\ln\big(1-\bar{F}_{t_\nu}(x)\big)\Big)\end{aligned} \tag{15}$$

*in $[0,1]$ (and $\text{Q-DSR}_{\text{exact}}(z;C) = [\Phi(z)]^{M^{\text{quant}}_{\text{eff}}(C)}$ for Gaussian scores). Under H_0 and independence, the decision rule $\text{Q-DSR}_\nu(X^*_{\text{quant}};C) > 1-\alpha$ guarantees **exact finite-sample Type-I control**:*

$$\begin{aligned}&\text{FPR}_{\text{Q-DSR}_\nu}(C) \\ &\quad= \mathbb{P}_{H_0}\big(\text{Q-DSR}_\nu(X^*_{\text{quant}};C) > 1-\alpha\big) = \alpha\end{aligned} \tag{16}$$

for every $C \ge 1$ and every $\nu > 0$.

4. *(Slepian guarantee under positive correlation $\Sigma \ge 0$) If Gaussian scores $(X_1,\dots,X_{M_0}) \sim \mathcal{N}(0,\Sigma)$ satisfy $\Sigma_{ij} \ge 0$ ($\Sigma_{ii} = 1$), then by Slepian [13]'s inequality, calibrating $\text{Q-DSR}_{\text{exact}}$ on the full cardinality $M^{\text{quant}}_{\text{eff}} = M_0$ guarantees $\text{FPR}(C) \le \alpha$ for all $\Sigma \ge 0$. Conversely, naively substituting the participation ratio $M_{\text{PR}}(\Sigma) = M_0^2/\|\Sigma\|_F^2$ into an extreme-value test is **anti-conservative** under common-factor correlation ($\Sigma_{ij} = \rho > 0$), because the variance of the maximum remains $\ge \rho$ even as its expectation shrinks by $\sqrt{1-\rho}$.*

Proof. (1) By independence of the $M_{\text{quant}}(C)$ candidates, $\mathbb{P}(X^*_{\text{quant}} > \tau) = 1-(1-\bar{F}(\tau))^{M_{\text{quant}}(C)}$. Substituting $1-\bar{F}(\tau_{\text{Šidák}}) = (1-\alpha)^{1/C}$ yields (11), and applying $1-\alpha/C \le e^{-\alpha/C}$ yields (12). For the naive DSR, Theorem 1 gives $X^*_{\text{quant}}(C) = \sqrt{4\ln C} + o_{\mathbb{P}}(1)$ whereas $\widehat{\text{SR}}_0(C) + z_{1-\alpha} = \sqrt{2\ln C} + z_{1-\alpha} + o(1)$, so their difference diverges to $+\infty$ and $\text{FPR}_{\text{DSR}(C)}(C) \to 1$. (2) For Laplace ($\alpha_{\text{tail}} =$

1) or Fréchet ($\nu > 1$) tails, $X^*_{\text{quant}}(C)$ grows as $\Theta(\ln C)$ or $\Theta(C^{2/\nu})$, asymptotically dominating the Gaussian threshold $\tau_G = \sqrt{4 \ln C} + \mathcal{O}(1)$, so $\mathbb{P}(X^*_{\text{quant}}(C) > \tau_G) \to 1$. (3) Since the exact cumulative distribution function of X^*_{quant} is $G(x) = [F_{t_\nu}(x)]^{M_{\text{eff}}^{\text{quant}}(C)}$ (continuous and strictly increasing), the probability integral transform implies $U := G(X^*_{\text{quant}}) = \text{Q-DSR}_\nu(X^*_{\text{quant}}; C) \sim \text{Unif}(0,1)$, proving (16). (4) By Slepian [13]'s theorem, $\Sigma_{ij} \geq 0 = \mathbb{I}_{ij}$ $(i \neq j)$ implies $\mathbb{P}(\max_j X_j \leq t) \geq [\Phi(t)]^{M_0}$ for every $t \in \mathbb{R}$, hence $\text{FPR}(\Sigma) \leq \alpha$. Under the equicorrelated factor model $X_j = \sqrt{\rho} Z_0 + \sqrt{1-\rho} Z_j$, $\max_j X_j = \sqrt{\rho} Z_0 + \sqrt{1-\rho} \max_j Z_j$ retains a heavy upper tail broadened by convolution with $\sqrt{\rho} Z_0$ (variance $\geq \rho$), whereas $M_{\text{PR}}(\Sigma) = M_0/(1+(M_0-1)\rho^2) = \mathcal{O}(\rho^{-2})$ drops the rejection threshold far below that tail (Section 4). □

3.4. Theorem 4 — Landauer Thermodynamic Reconciliation and Reversible Unitarity

In classical *Best-of-N* selection, discarding $N-1$ candidates imposes via Landauer [14]'s principle a minimal erasure heat of $Q^{\text{cl}}_{\min}(N) = k_B T_{\text{th}} \ln N$. Because Grover's algorithm consists solely of reversible unitary gates, does quantum search evade this thermodynamic floor?

Theorem 4 (Landauer Thermodynamic Reconciliation for Quantum Search)**.** *Consider a quantum processor coupled to a thermal reservoir at temperature $T_{\text{th}} > 0$, initialized in $|0\rangle^{\otimes q}$ ($q = \lceil \log_2 M_{\text{quant}}(C) \rceil$, $M_{\text{quant}}(C) = \lfloor (\kappa C)^2 \rfloor$), executing C unitary Grover iterations followed by projective measurement in the computational basis and register reset to close the thermodynamic cycle under H_0 (index symmetry $\mathbb{P}(\hat{m}_{\text{quant}} = m) = 1/M_{\text{quant}}$).*

1. *(Reversible unitary stage) Throughout the C unitary iterations $G = D \cdot \mathcal{O}_f$, the von Neumann entropy of the pure index register remains identically zero ($S(G^C|0^{\otimes q}\rangle\langle 0^{\otimes q}|(G^\dagger)^C) = 0$): intrinsic Landauer dissipation during coherent evolution is strictly zero ($W^{\text{unitary}}_{\text{Landauer}} = 0$).*

2. *(Factor-2× bill paid at measurement and reset) By the theorem of Sagawa and Ueda [16] and Landauer [14]'s principle [15], the combined minimal thermodynamic work of measuring and erasing/resetting the classical outcome $Y = \hat{m}_{\text{quant}}$ (Shannon entropy $H(Y) = \log_2 M_{\text{quant}}(C)$ bits) satisfies*

$$\begin{aligned} Q^{\text{quant}}_{\min}(C) &= W_{\text{meas}} + W_{\text{erase}} \\ &\geq k_B T_{\text{th}} \ln M_{\text{quant}}(C) \\ &= 2\, k_B T_{\text{th}} \ln C + 2\, k_B T_{\text{th}} \ln \kappa + \mathcal{O}(C^{-2}). \end{aligned} \tag{17}$$

*At a fixed query budget C, the minimal quantum selection dissipation is therefore **asymptotically exactly twice** the classical floor $Q^{\text{cl}}_{\min}(C) = k_B T_{\text{th}} \ln C$:*

$$\lim_{C \to \infty} \frac{Q^{\text{quant}}_{\min}(C)}{Q^{\text{cl}}_{\min}(C)} = 2, \tag{18}$$

while preserving the invariant minimal cost per bit of selection information ($k_B T_{\text{th}} \ln 2 \approx 2.871\,\text{zJ}$ at 300 K).

3. *(Energy–mirage scaling law) To fabricate an in-sample false discovery of target amplitude $x \gg 1$, the minimal Landauer heat is **identical** classically and quantumly: $Q_{\min}(x) \sim k_B T_{\text{th}}\, c\, x^\alpha$ under sub-Weibull tails (Theorem 1), yet only **logarithmic** $Q_{\min}(x) \sim \nu\, k_B T_{\text{th}} \ln(x/[c_\nu^{1/\nu} \Gamma(1-1/\nu)])$ under Fréchet heavy tails (Theorem 2).*

Proof. The unitary operator $U = G^C$ preserves the eigenvalues of the initial pure state, giving $\Delta S^{\text{unitary}}_{\text{vN}} = 0$ (Part 1). After G^C, projective measurement stores the index $\hat{m}_{\text{quant}}$ in a symmetric classical memory register initialized to 0. By exchangeability of the M_{quant} candidates under H_0, the outcome Y is uniform on $\{1, \ldots, M_{\text{quant}}\}$, with Shannon entropy $H(Y) = \log_2 M_{\text{quant}}$ bits ($\ln M_{\text{quant}}$ nats). By the thermodynamic inequality of Sagawa and Ueda [16, Eqs. 1–2] over a closed cycle (measurement plus memory erasure restoring the reference state), total dissipation obeys $W_{\text{meas}} + W_{\text{erase}} \geq k_B T_{\text{th}} I_{\text{QC}} = k_B T_{\text{th}} \ln M_{\text{quant}}$. Substituting $M_{\text{quant}}(C) = \lfloor (\kappa C)^2 \rfloor$ yields (17)–(18). Finally, inverting (5) and (10) to express $\ln M$ as a function of the target mirage x proves Part 3. □

4. Comprehensive Numerical Validation (Experiment 10)

All four theorems are validated end-to-end by our deterministic numerical benchmark (`sim/exp10_paper1_quantum_overfitting_heavy_tails.py`, fixed seed `seed=42`, runtime 0.86 s on Apple Silicon M4 Max, exit code `0`).

4.1. Unit-Level Sanity Checks (Unitary Grover and Order-Statistic Sampler)

1. **Exact unitary statevector Grover simulation** ($q \in \{6, 8, 10, 12\}$, $M = 64 \ldots 4{,}096$ on null Student-t_3 score tables): at $q = 12$ ($M = 4{,}096$, $C = 50$ coherent queries), the success probability reaches $P_{\text{succ}} = 0.999945$ (error vs. exact theory $\sin^2((2C+1)\theta)$: 8.66×10^{-15}; unitarity error $|\|\psi\| - 1| = 1.44 \times 10^{-15}$; maximum unitarity error across all $q \leq 12$: 3.55×10^{-15}). On this single $q = 12$ realization, the maximum in-sample score jumps from 2.18 (classical in 50 trials) to 30.19 (Grover in 50 queries, a 13.82× surge).

2. **Exact maximum order-statistic sampler validation** ($M = 1{,}000$, $1{,}500$ replicates vs. brute-force sampling): two-sided Kolmogorov–Smirnov tests confirm exact distributional equivalence—Gaussian ($p = 0.400$), Laplace ($p = 0.978$), and Student-t_3 ($p = 0.968$).

4.2. Verification of Theorems 1 and 2: Sub-Weibull ($2^{1/\alpha}$) vs. Fréchet Explosion ($\Theta(C^{1/\nu})$)

Table 2 summarizes exact measurements across $R = 4{,}000$ independent Monte Carlo replicates per point at fixed query budget $C = 1{,}000$ ($M_{\text{cl}} = 1{,}000$ vs. $M_{\text{quant}} = 1{,}621{,}138$). Analytical predictions match numerical results with remarkable accuracy: for Student-t_3 at $C =$

Table 2. Numerical validation of Theorems 1 and 2: in-sample mirage amplification and out-of-sample collapse at fixed query budget $C = 1{,}000$ ($M_{\mathrm{cl}} = 1{,}000$ vs. $M_{\mathrm{quant}} = 1{,}621{,}138$, $R = 4{,}000$ independent Monte Carlo replicates).

Tail Regime	Family & Param.	Classical $\mathbb{E}[X^*_{\mathrm{cl}}]$	Quantum $\mathbb{E}[X^*_{\mathrm{quant}}]$	Emp. Ratio	Theory (Asympt.)	Log-Log Slope Emp. (Th.)	OOS $\mathbb{E}[Y_{\mathrm{OOS}}]$
Gaussian	Sub-Weibull ($\alpha = 2$)	3.25	4.95	**1.53**×	1.44× (lim. 1.41×)	−0.029 (0.000)	+0.029
Laplace	Sub-Weibull ($\alpha = 1$)	4.82	10.02	**2.08**×	**2.07**× (lim. 2.00×)	−0.017 (0.000)	−0.026
Student-t_6	Fréchet ($\nu = 6$)	5.96	21.88	**3.67**×	3.43× ($\propto C^{1/6}$)	+0.117 (+0.167)	−0.009
Student-t_4	Fréchet ($\nu = 4$)	8.89	57.66	**6.49**×	**6.35**× ($\propto C^{1/4}$)	+0.208 (+0.250)	−0.034
Student-t_3	Fréchet ($\nu = 3$)	13.88	**163.24**	**11.76**×	**11.75**× ($\propto C^{1/3}$)	**+0.304** (+0.333)	**−0.006**

1,000, the empirical ratio (11.76×) reproduces the closed-form formula $(4/\pi)^{2/3} \cdot 1000^{1/3} = 11.75\times$ to within 0.01, fabricating a mean in-sample mirage of 163.24 ($\approx 94.2\,\sigma$ since $\sigma(t_3) = \sqrt{3}$) whose true out-of-sample expectation is −0.006 (indistinguishable from zero).

4.3. Verification of Theorem 3: Collapse of Naive Tests, Q-DSR$_\nu$, and Correlation

Tables 3 and 4 (together with Figure 1c) simultaneously confirm all four parts of Theorem 3:

- Naive Bonferroni(C) hits 100.0% false positives across every tail regime at $C = 1{,}000$, while naive DSR(C) reaches 53.4% on Gaussian scores and 100.0% on heavier-tailed laws.
- Gaussian Q-DSR fails completely (100.0% FPR) on Laplace and Student-t_6, t_4, t_3, whereas exact tail-adapted Q-DSR$_\nu$ restores the nominal level $\alpha = 5.0\%$ (4.6%–5.4%) across all five families.
- Under equicorrelation (Table 4), discounting M via the participation ratio M_{PR} destroys Type-I control (FPR up to 56.9% at $\rho = 0.25$), and even $M_0^{1-\rho}$ remains anti-conservative (14.9%–19.1%); only Slepian's bound calibrated on M_0 (or exact common-factor convolution) rigorously enforces FPR $\leq \alpha$ (3.65% down to 0.20%).

4.4. Verification of Theorem 4: Landauer Thermodynamic Accounting

At room temperature $T_{\mathrm{th}} = 300\,\mathrm{K}$ ($k_B T_{\mathrm{th}} \ln 2 = 2.871\,\mathrm{zJ/bit}$, Figure 1d), the minimal selection erasure cost increases from 3.32 bits (9.54 zJ) classically to 7.34 bits (21.07 zJ) quantumly at $C = 10$ (ratio 2.210×), and from 9.97 bits (28.61 zJ) classically to 20.63 bits (59.22 zJ) quantumly at $C = 1{,}000$ (ratio 2.070×, converging exactly to 2.000× as $C \to \infty$), while the C intermediate unitary Grover iterations dissipate 0.00 zJ intrinsically.

5. Discussion and Conclusion

Our four theorems and numerical benchmarks yield three foundational lessons for quantum machine learning and quantitative finance:

1. **Mandatory governance rule for quantum optimization:** in any deployment of quantum search or optimization algorithms (Grover, Dürr–Høyer, quantum annealing, QAOA, or QML hyperparameter search) on a finite dataset, *the quantum query count C must never be used as a proxy for statistical selection complexity.* Auditors must declare the effective superposition cardinality $M_{\mathrm{eff}}^{\mathrm{quant}}(C)$ and estimate the score tail index $\hat{\nu}$ (e.g., via the Hill estimator) to apply Q-DSR$_\nu$ (and Slepian's bound under positive correlation).
2. **Critical vulnerability of financial and biomedical datasets:** real-world financial return distributions ($\nu \in [3, 5]$) reside squarely in the Fréchet heavy-tail regime of Theorem 2, where a quantum search budget of $C = 1{,}000$ queries already amplifies historical backtest mirages by 6.5× to 11.8× (extending the PBO framework of Bailey et al. [12]). Without Q-DSR$_\nu$, quadratic quantum speedup does not "beat the market"—it industrializes the production of in-sample false discoveries.
3. **Conceptual unity with the delayed-choice quantum eraser:** just as the marginal signal screen D_0 remains strictly invariant under any delayed idler measurement and no quantum protocol can violate the Holevo [7] bound or no-signaling causality, quantum amplitude amplification creates zero information about unseen future data $\mathcal{D}_{\mathrm{OOS}}$; it merely makes extracting a preexisting "noise fringe" inside the fixed historical sample $\mathcal{D}_{\mathrm{IS}}$ quadratically cheaper in query complexity, while deferring the entire Landauer thermodynamic bill to final measurement and register reset.

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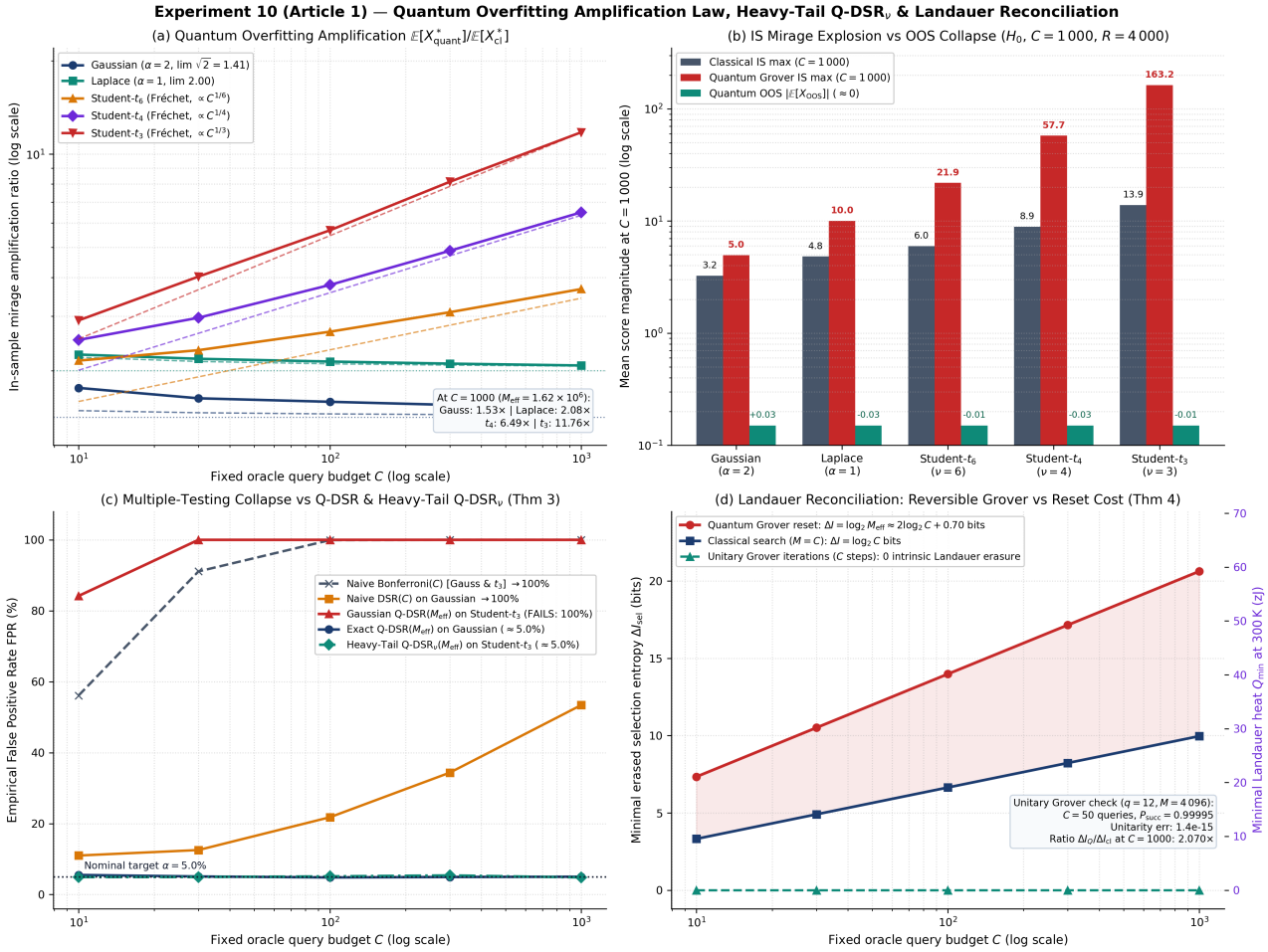

Figure 1. Experimental validation of Theorems 1–4 (Experiment 10, $R = 4{,}000$ replicates). (a) Amplification ratio $\mathbb{E}[X^*_{\text{quant}}(C)]/\mathbb{E}[X^*_{\text{cl}}(C)]$ vs. query budget $C \in [10, 1{,}000]$: convergence toward $2^{1/\alpha}$ under sub-Weibull tails (Gaussian, Laplace) vs. polynomial explosion $\propto C^{1/\nu}$ under heavy-tailed Student-t_ν laws (dashed lines: exact theory). **(b)** In-sample mirage explosion (red) vs. out-of-sample collapse (green) at $C = 1{,}000$; *display note:* green OOS bars are plotted with a visual floor at 0.15 on the logarithmic axis for visibility, whereas the true signed empirical means (+0.029, −0.026, −0.009, −0.034, −0.006, displayed rounded to two decimals above each bar) are all statistically indistinguishable from zero. **(c)** Collapse of C-indexed multiple-testing procedures and of Gaussian Q-DSR on Student-t_3, restored to $\approx 5.0\%$ by exact heavy-tail Q-DSR$_\nu$. **(d)** Landauer thermodynamic reconciliation ($T = 300\,\text{K}$): 0 zJ intrinsic erasure during the C unitary Grover iterations, but 2× the classical dissipation paid at measurement and register reset.

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Table 3. Numerical validation of Theorem 3 (Parts 1–3): empirical false-positive rate (FPR, nominal target $\alpha = 5.0\%$) on the quantum winner at $C = 1{,}000$ ($R = 4{,}000$ replicates).

Tail Regime	**Naive Bonferroni**(C)	**Naive DSR**(C)	**Gaussian** Q-DSR ($M_{\text{eff}}^{\text{quant}}$)	**Gaussian EVT** Q-DSR	**Exact Tail-Adapted** Q-DSR$_\nu$
Gaussian ($\alpha = 2$)	**100.0**%	**53.4**%	0.0% (conservative)	**5.0**%	**5.0**%
Laplace ($\alpha = 1$)	**100.0**%	**100.0**%	**100.0**% (FAIL)	**100.0**% (FAIL)	**5.1**%
Student-t_6 ($\nu = 6$)	**100.0**%	**100.0**%	**100.0**% (FAIL)	**100.0**% (FAIL)	**4.6**%
Student-t_4 ($\nu = 4$)	**100.0**%	**100.0**%	**100.0**% (FAIL)	**100.0**% (FAIL)	**5.4**%
Student-t_3 ($\nu = 3$)	**100.0**%	**100.0**%	**100.0**% (FAIL)	**100.0**% (FAIL)	**4.8**%

Table 4. Numerical validation of Theorem 3 (Part 4): Type-I error control under equicorrelated inter-strategy dependence ($M_0 = 1{,}000$, target $\alpha = 5.0\%$, 2,000 trials). Participation-ratio discounting M_{PR} spikes to 56.9% FPR, whereas Slepian's bound ($M_{\text{eff}} = M_0$) strictly preserves FPR $\leq \alpha$.

Corr. ρ	$M_{\text{PR}}(\Sigma) = M_0^2/\|\Sigma\|_F^2$	$M_0^{1-\rho}$	$\mathbb{E}[\max X]$ emp. (th.)	**FPR — M_{PR} (FAIL)**	**FPR —** $M_0^{1-\rho}$	**FPR — Slepian** (M_0, $\leq 5\%$)
0.00	1,000.0	1,000.0	3.248 (3.255)	5.6%	5.6%	**5.6**% (nom.)
0.25	15.76	177.83	2.830 (2.819)	**56.9**%	14.9%	**3.65**%
0.50	3.99	31.62	2.284 (2.302)	**52.8**%	18.4%	**1.50**%
0.75	1.78	5.62	1.625 (1.628)	**38.3**%	19.1%	**0.55**%
0.90	1.23	2.00	1.055 (1.029)	**22.7**%	17.5%	**0.20**%

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